Asymptotics of Rational Solutions of the Inhomogeneous Painlevé-II Equations

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Abstract:

The inhomogeneous Painlevé-II equation with constant parameter α has a unique rational solution exactly when α is an integer. This talk concerns the asymptotic behavior of this rational solution in the limit of large $|\alpha|$. Numerical calculations of Clarkson and Mansfield have shown that the poles and zeros of these rational functions seem to be confined to a triangular region in the complex plane, forming a regular lattice pattern within this region. Using the representation of the rational solutions in terms of a matrix Riemann-Hilbert problem and applying the Deift-Zhou steepest descent method to the latter problem, we prove the existence of an asymptotically confining region for the poles and zeros, a region which is not exactly a triangle but is rather a curvilinear triangle with equal corner angles of $2\pi/5$. We also can give asymptotic formulae for the rational solutions in all regions of the complex plane: inside the region, outside the region, near one of the edges of the region, and near one of the corners of the region. Asymptotics in the interior are given in terms of elliptic functions, similar to the Boutroux ansatz that holds for general (non-rational) solutions of Painlevé-II for large arguments. Near the corners, the Hamiltonian of the tritronquée solution of the Painlevé-I equation describes the asymptotic behavior of the rational functions. This is joint work with R. Buckingham.